Network Structural Equation Models for Causal Mediation and Spillover Effects

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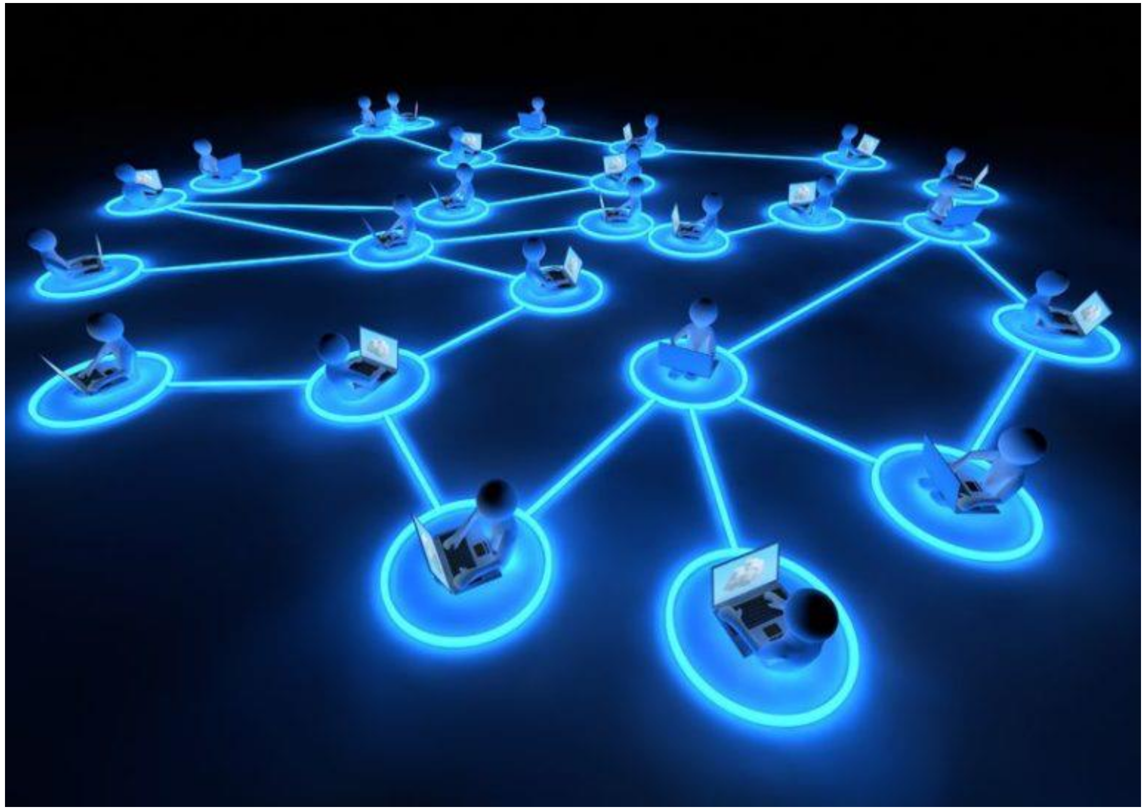
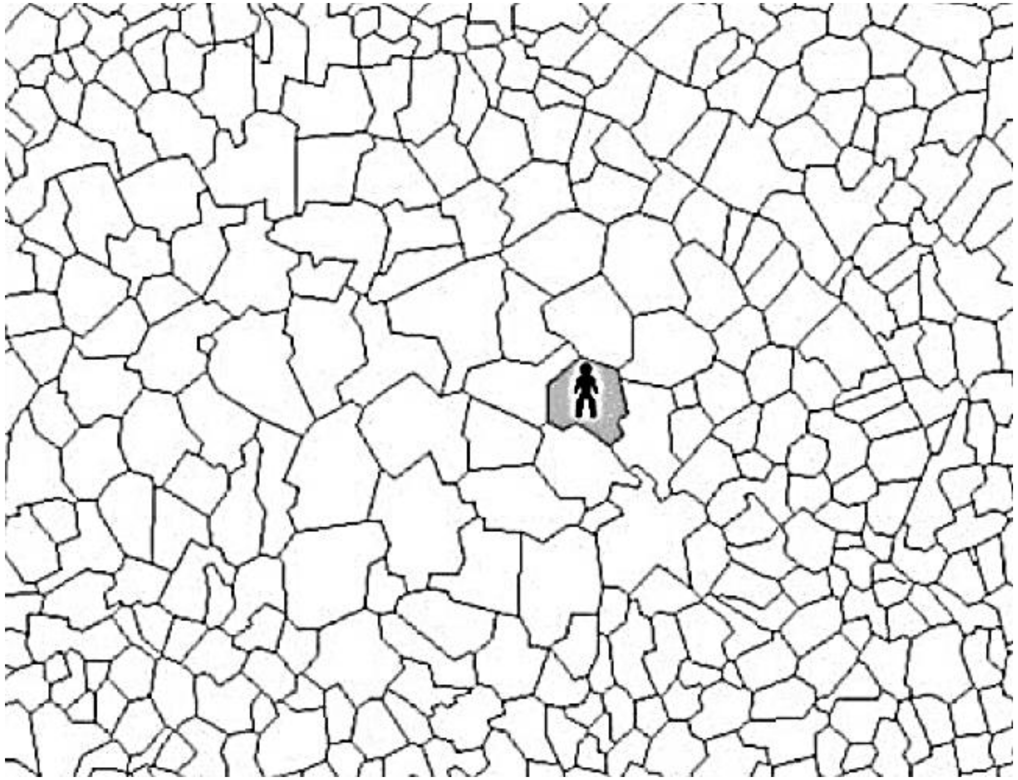
We are socially and/or Environmentally linked

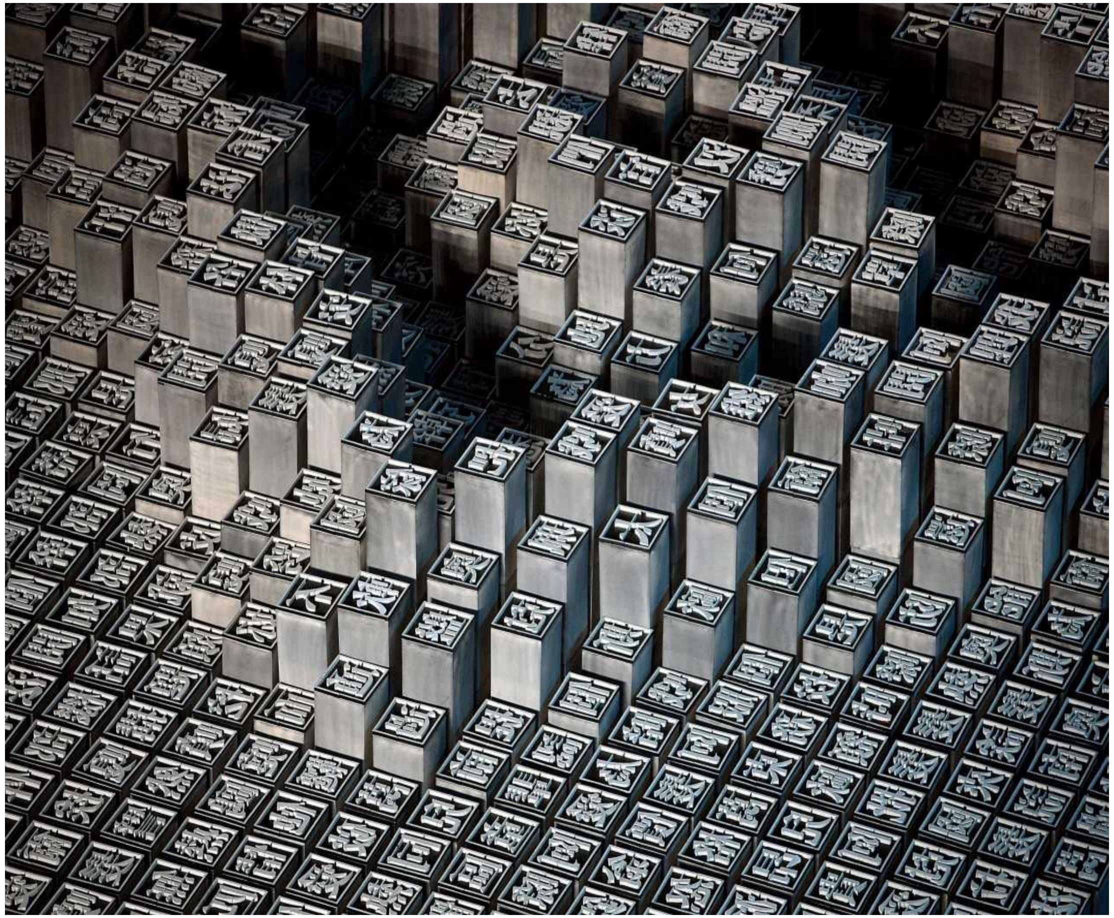
Linked by social networks



Linked by shared environments



A systematic and synergistic approach



Change in an element in the system may cause changes in other elements in the system.

A system of many elements that are socially or environmentally linked

Vertex/node/subject/area

Edge/connectivity

$\mathcal{H} = \{1, \cdots, N\}$

$(Y_i, M_i, A_i, C_i), i \in \mathcal{H}$

Y: outcome

M: Mediator

A: Exposure

C: Confounding

Example 1: Friend Network

- $\mathcal{H} = \{all\ undersgrads\}$

1

A: Enroll AI courses (yes/no)

2

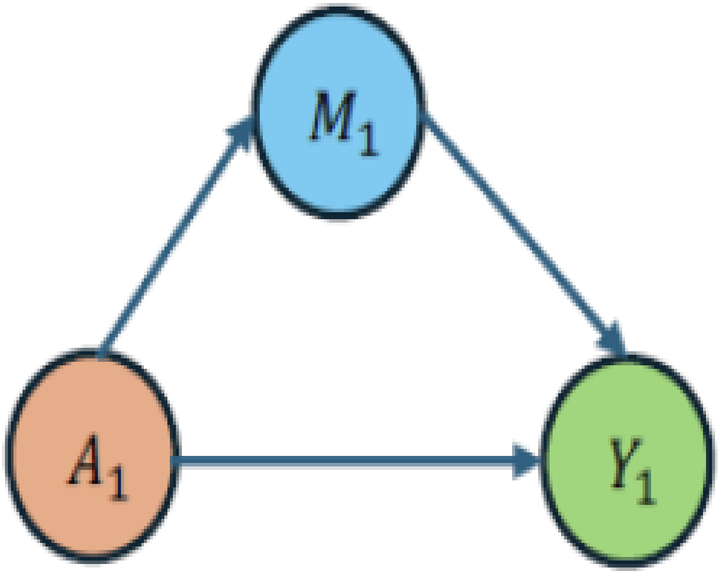
M: Take a parttime job (yes/no)

3

Y: Financial Burden

4

C: Sex, GPA, SES, etc.



Directed Acyclic Graph (DAG)

Example 2: Business Network

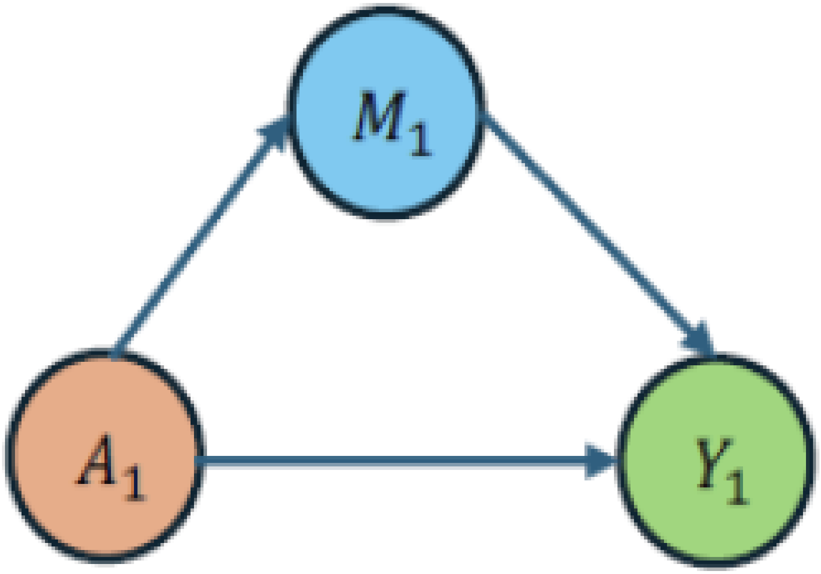
- $\mathcal{H} = \{all\ export - centric\ companies\ in\ a\ country\}$

A: Goods-specific tariff from USA (percent)

M: Product price adjustment

Y: Annual revenue

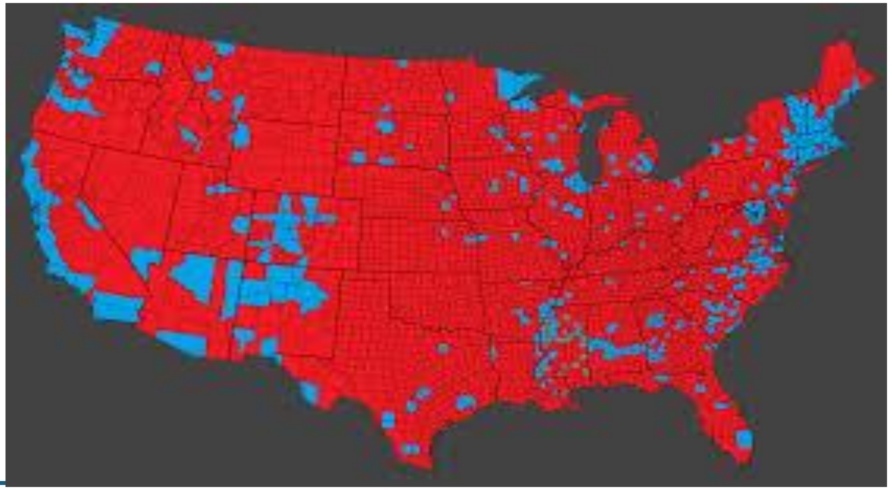
C: Product type, rebate policy, etc.



Directed Acyclic Graph (DAG)

Example 3: Infectious Disease Network

$\mathcal{H} = \{all\ counties\ in\ the\ USA\}$

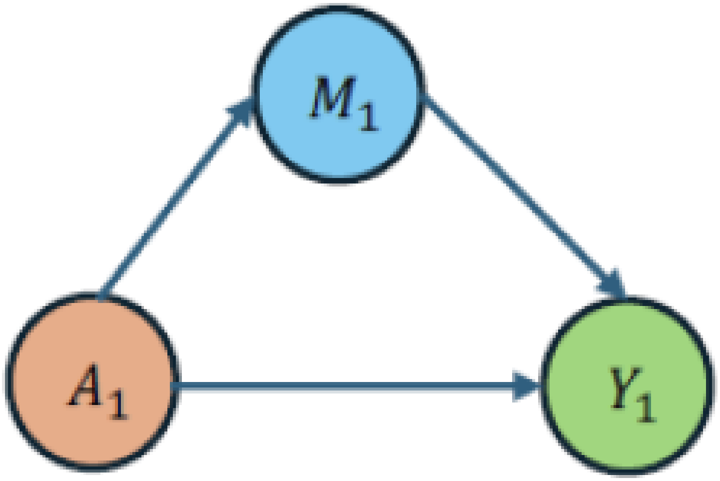


A: County-level political party affiliation (Blue/Red)

M: County-level vaccination Compliance

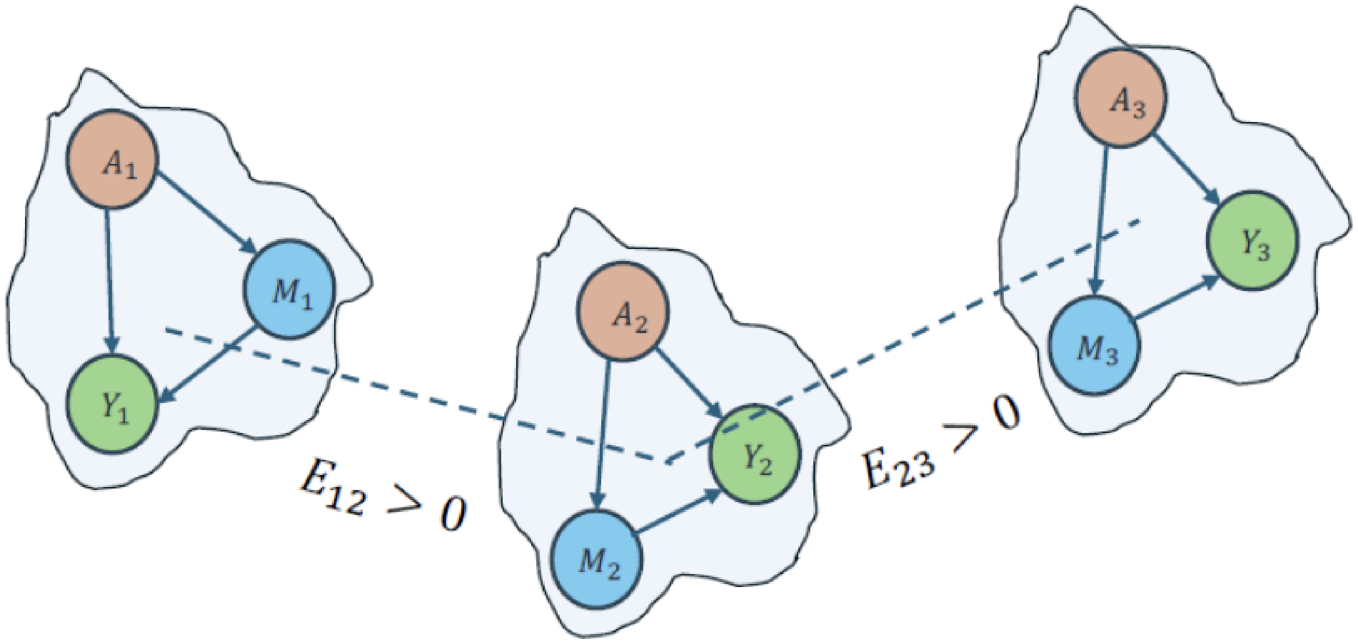
Y: County-level number of deaths, hospitalizations, or confirmed cases

C: Sex, race, age, socioeconomic status, etc.



Directed Acyclic Graph (DAG)

A Network of Units that are linked

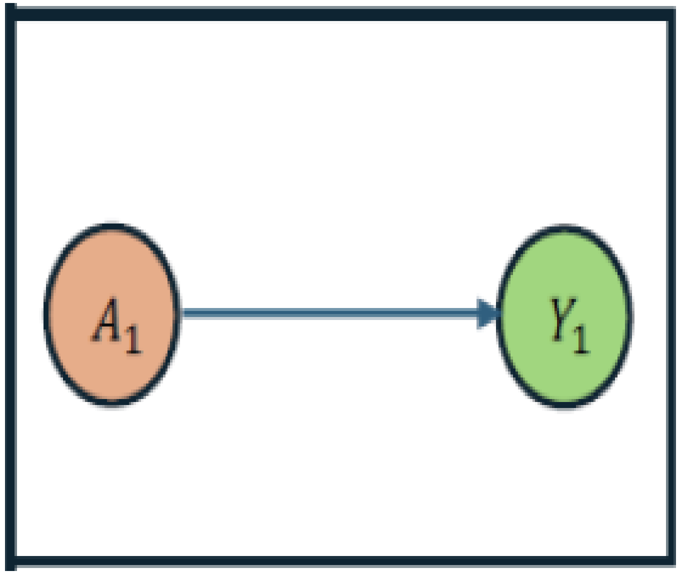


In addition to your own plan of course selection, your plan may be influenced by your friend's plan.

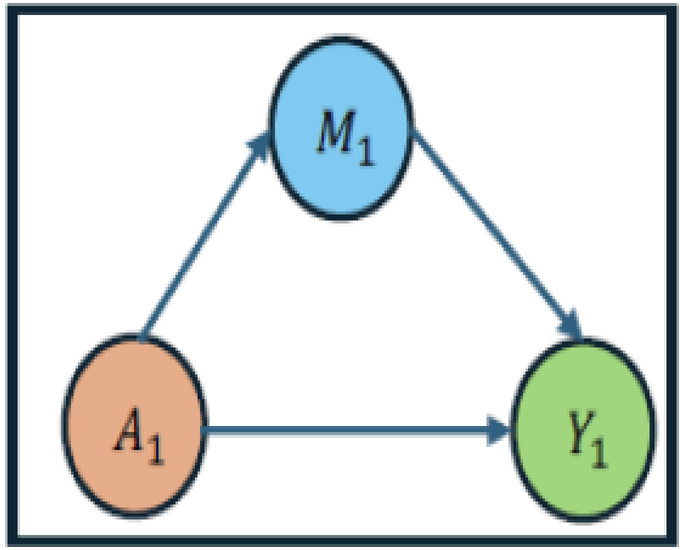
An areal network of counties that are spatially connected

3109 continental counties, each having a county-level DAG

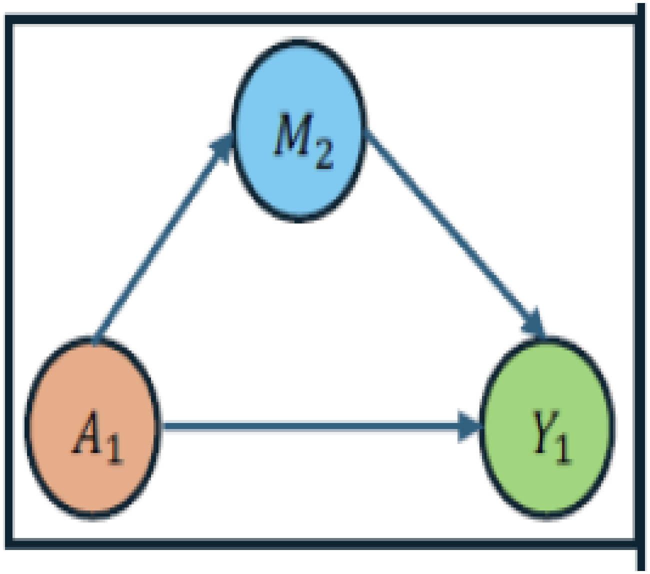
What do we want to study?
Self-Initiated (indogenous) Effects (3 kinds)



τ_1 (A→ Y)



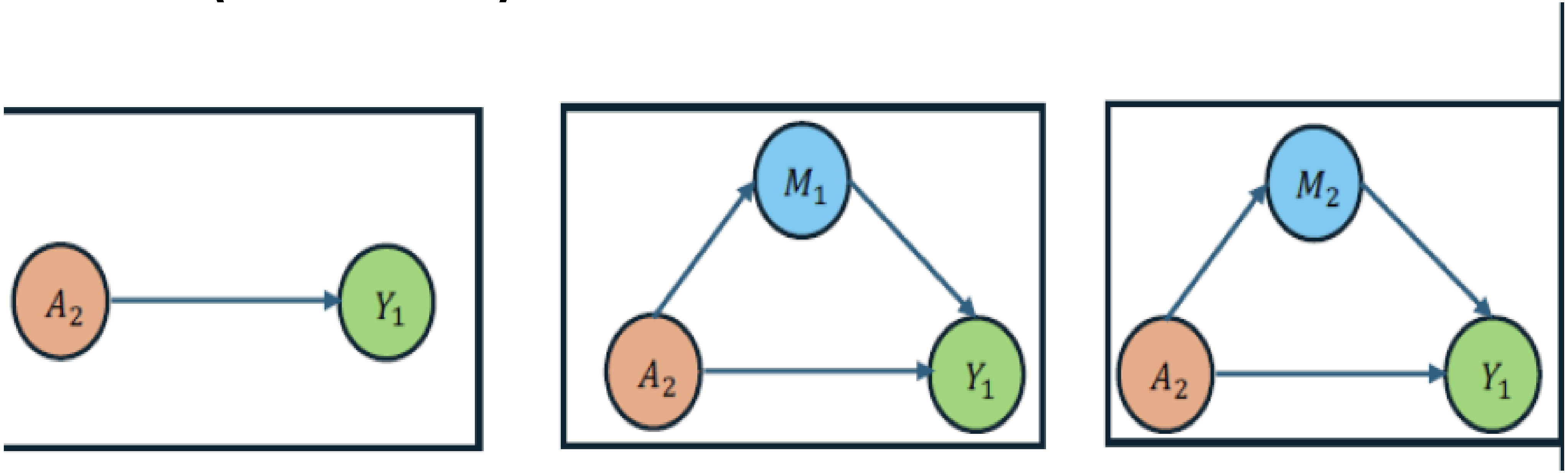
τ_2 (A $\rightarrow M \rightarrow Y$)



τ_3 (A $\rightarrow M^f \rightarrow Y$)

Your own plan of course selection affects your own outcomes directly or via your own mediator or your friend's mediator

Spillover Effects: Friend-Initiated (exogenous) Effects (3 kinds)



$\tau_4(A^f \rightarrow Y)$ $\tau_5(A^f \rightarrow M \rightarrow Y)$ $\tau_6(A^f \rightarrow M^f \rightarrow Y)$

Your friend’s study plan influences your outcomes via your own mediator or your friend’s mediator

Connectivity

Spatial Adjacency Matrix E

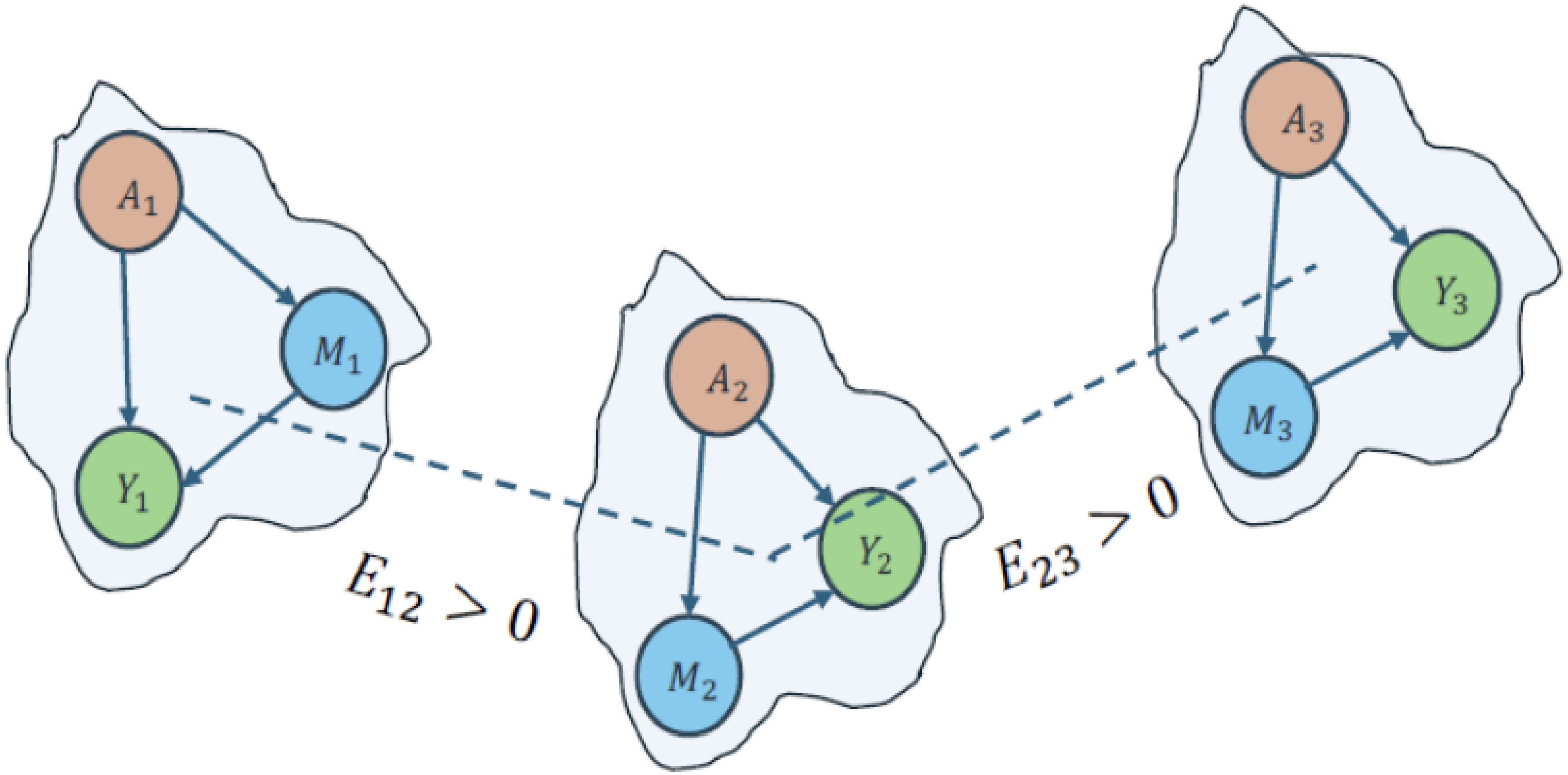
- Constructed using the Queen contiguity rule:
  - $E_{ij} > 0$: Polygons i and j share an edge or vertex.
  - $E_{ii} = 1$: Self-connectivity.
  - $E_{ij} = 0$: Non-neighboring polygons.
- Captures first-order Markov spatial dependencies.

Neighborhood

- **Immediate Neighbors ($\mathcal{N}_i^\dagger$):** Polygons directly connected to i: $\mathcal{N}_i^\dagger = \{j \mid j \neq i, E_{ij} > 0\}$.
- **Extended Neighborhood ($\mathcal{N}_i$):** Includes i and its immediate neighbors: $\mathcal{N}_i = \mathcal{N}_i^\dagger \cup \{i\}$.
- **Second-Degree Neighbors ($\mathcal{N}_i^\ddagger$):** Indirect neighbors of i via $\mathcal{N}_i^\dagger$:

$$\mathcal{N}_i^\ddagger = \{k \mid k \neq i, k \notin \mathcal{N}_i^\dagger, E_{jk} > 0 \text{ for } j \in \mathcal{N}_i^\dagger\}.$$

Toy Example



Target County 1: $\mathcal{N}_1^{\dagger} = \{2\}$, $\mathcal{N}_1^{\ddagger} = \{3\}$.

Outcome Model

Total exposure from 1-degree friends

Total mediation from 1-degree friends

Model for Outcome Y

The outcome Y_i for unit i is modeled as:

$$Y_i = \beta_0 + \beta_1 A_i + \beta_2 S_{1i}(\boldsymbol{A}_{\mathcal{N}_i^\dagger}) + \beta_3 M_i + \beta_4 S_{1i}(\boldsymbol{M}_{\mathcal{N}_i^\dagger})$$

$$+\beta_5^T \boldsymbol{C}_i + \beta_6^T \boldsymbol{S}_{1i}(\boldsymbol{C}_{\mathcal{N}_i^\dagger}) + E_{i.}^T \boldsymbol{b}^Y + \epsilon_i^Y,$$

where:

- $\epsilon^Y \sim \text{MVN}(0, \sigma_y^2 \mathcal{I}_{N\times N})$: Residuals.
- $\boldsymbol{b}^Y \sim \text{MVN}(0, \sigma_{b^Y}^2 \mathcal{I}_{N\times N})$: Random effects.
- $E_{i.}^T$: i^{th} row of spatial adjacency matrix E.

Mediator Model

Model for Mediator M

The mediator M_i for unit i is modeled as:

$$M_i = \gamma_0 + \gamma_1 A_i + \gamma_2 S_{1i}(\boldsymbol{A}_{\mathcal{N}_i^\dagger}) + \gamma_3^T \boldsymbol{C}_i + \gamma_4^T \boldsymbol{S}_{1i}(\boldsymbol{C}_{\mathcal{N}_i^\dagger}) + E_{i.}^T \boldsymbol{b}^M + \epsilon_i^M,$$

where:

- $\boldsymbol{\epsilon}^M \sim \text{MVN}(0, \sigma_m^2 \mathcal{I}_{N\times N})$: Residuals.
- $\boldsymbol{b}^M \sim \text{MVN}(0, \sigma_{bM}^2 \mathcal{I}_{N\times N})$: Random effects.
- $E_{i.}^T$: i^{th} row of spatial adjacency matrix E.

Exposure Model

Model for Exposure A

The exposure A_i for unit i is modeled as:

$$A_i = \alpha_0 + \alpha_1 \boldsymbol{C}_i + \alpha_2 \boldsymbol{S}_{1i}(\boldsymbol{C}_{\mathcal{N}_i^\dagger}) + \epsilon_i^A,$$

where:

- $\epsilon^A \sim \text{MVN}(0, \sigma_A^2 \mathcal{I}_{N \times N})$.

Identifiability Conditions & Estimation Framework

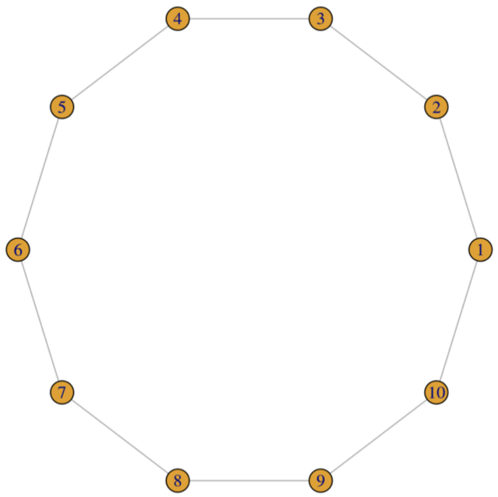
- Maximum Likelihood Estimation
- Six causal mediation estimands τ_1 *to* τ_6 are estimated by the plug-in method as each
$$\tau = \tau(\alpha, \beta, \gamma)$$
- Large-sample theory for non-iid samples (Sweeting et al. 1980, AoS; Mardia et al. 1984, BKA)

- Extended sequential ignorability conditions are proposed to identify the six causal mediation estimands under the counterfactual framework

(Imai et al. 2010, Stat Sci; Ibens, Am Econ Rev)

Simulation Experiment

- Network size N: 100, 200, 800
- Adjacency matrix E: symmetric 1-dependence
- Binary exposure A ~ Ber(0.5)
- Continuous mediator M and Y are generated by REN-SEM.



- **General Results:**
  - Estimation biases for all six estimands are minimal and decrease with larger sample sizes.
  - RRMSE also declines with increasing N, validating the statistical reliability of MLE.
  - Coverage probabilities approach the nominal level of 0.95, confirming the validity of confidence intervals.

Size(N)	Effects	Actual	Bias	RRMSE	ESE	ASE	CP
100	τ_1	1.50	-0.028	0.341	0.443	0.426	0.950
	τ_2	2.40	-0.095	0.259	0.458	0.83	0.960
	τ_3	0.18	-0.005	0.741	0.132	0.134	0.940
	τ_4	0.80	-0.003	0.790	0.626	0.620	0.948
	τ_5	1.08	-0.042	0.651	0.653	0.674	0.950
	τ_6	0.80	-0.026	0.433	0.321	0.324	0.955
200	τ_1	1.50	-0.008	0.202	0.303	0.297	0.948
	τ_2	2.40	0.005	0.140	0.340	0.341	0.966
	τ_3	0.18	-0.008	0.493	0.088	0.090	0.940
	τ_4	0.80	0.034	0.565	0.451	0.437	0.942
	τ_5	1.08	-0.017	0.425	0.459	0.477	0.964
	τ_6	0.80	-0.022	0.285	0.227	0.224	0.934
800	τ_1	1.50	0.005	0.098	0.148	0.148	0.942
	τ_2	2.40	0.008	0.071	0.170	0.170	0.946
	τ_3	0.18	0.002	0.250	0.046	0.046	0.958
	τ_4	0.80	0.003	0.275	0.220	0.218	0.952
	τ_5	1.08	0.006	0.217	0.234	0.239	0.958
	τ_6	0.80	0.007	0.136	0.108	0.110	0.954

Application: Party Affiliation, Vaccination Hesitance, and COVID-19 Mortality

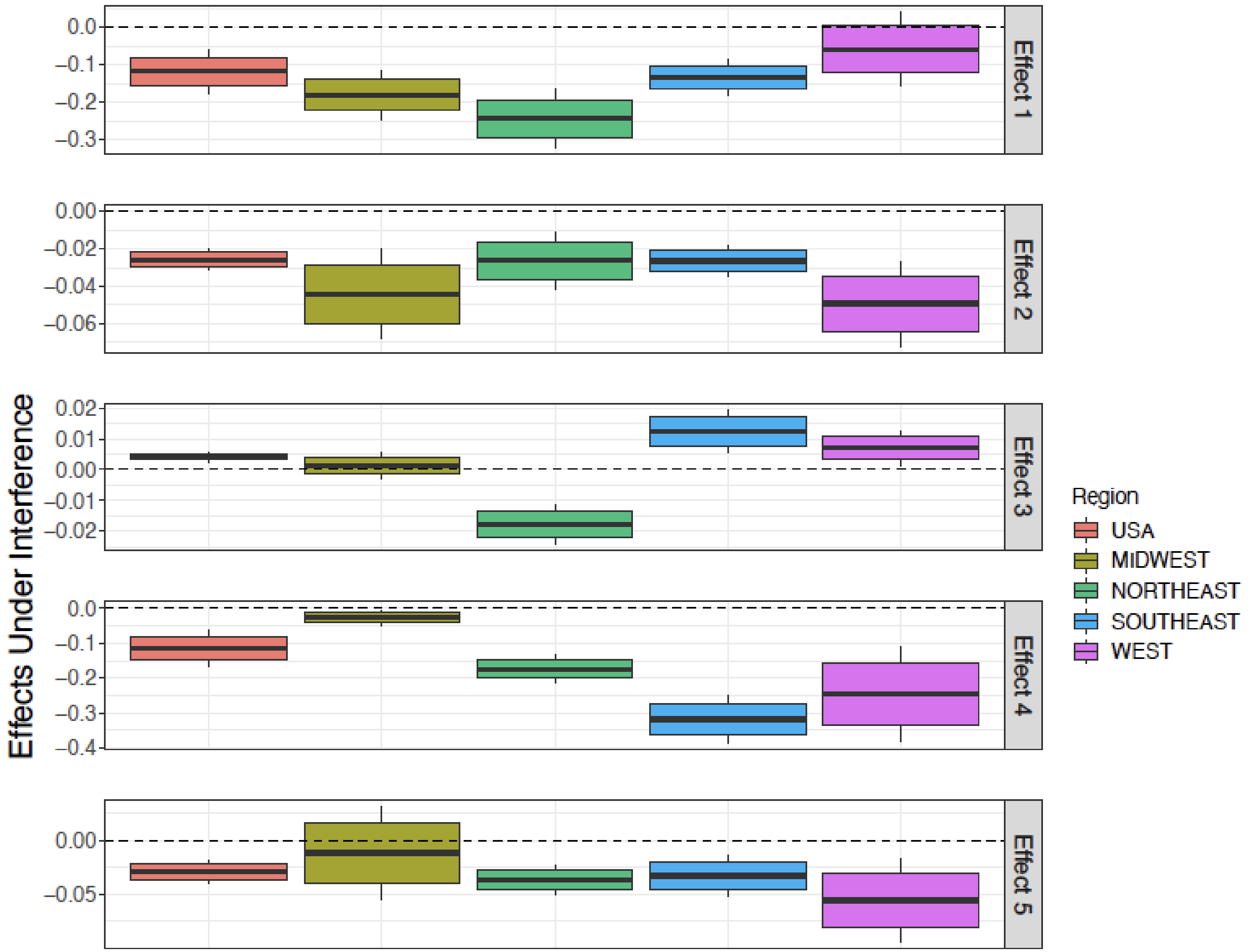
- Analysis stratified into Midwest, Northeast, Southeast, and West regions, as well as nationwide.

- Reginal stratification highlights heterogeneity in the impact of political affiliation (PA) on COVID-19 mortality mediated by vaccine hesitancy (VH).

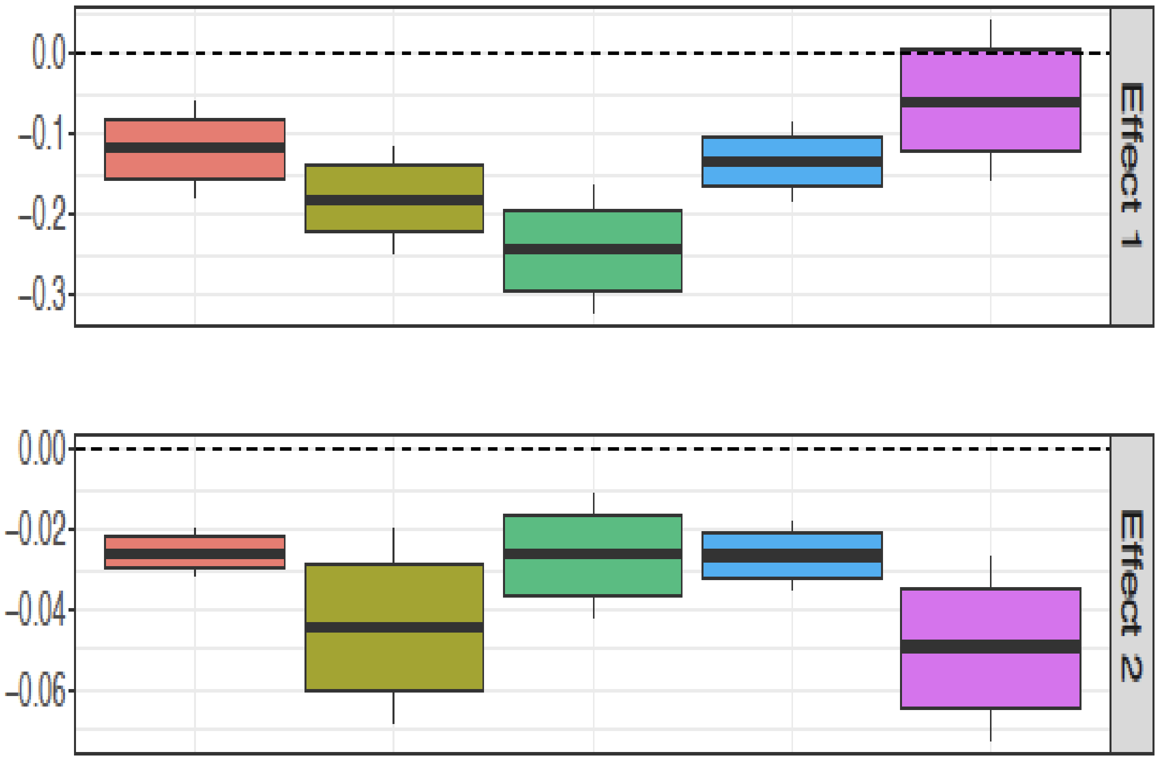
- Effects evaluated for counterfactual exposure change:

  (self, neighbor) = (0,0) (Republican) to (1,1) (Democratic)

- Other counterfactual exposure changes are also possible, e.g. (1,0).

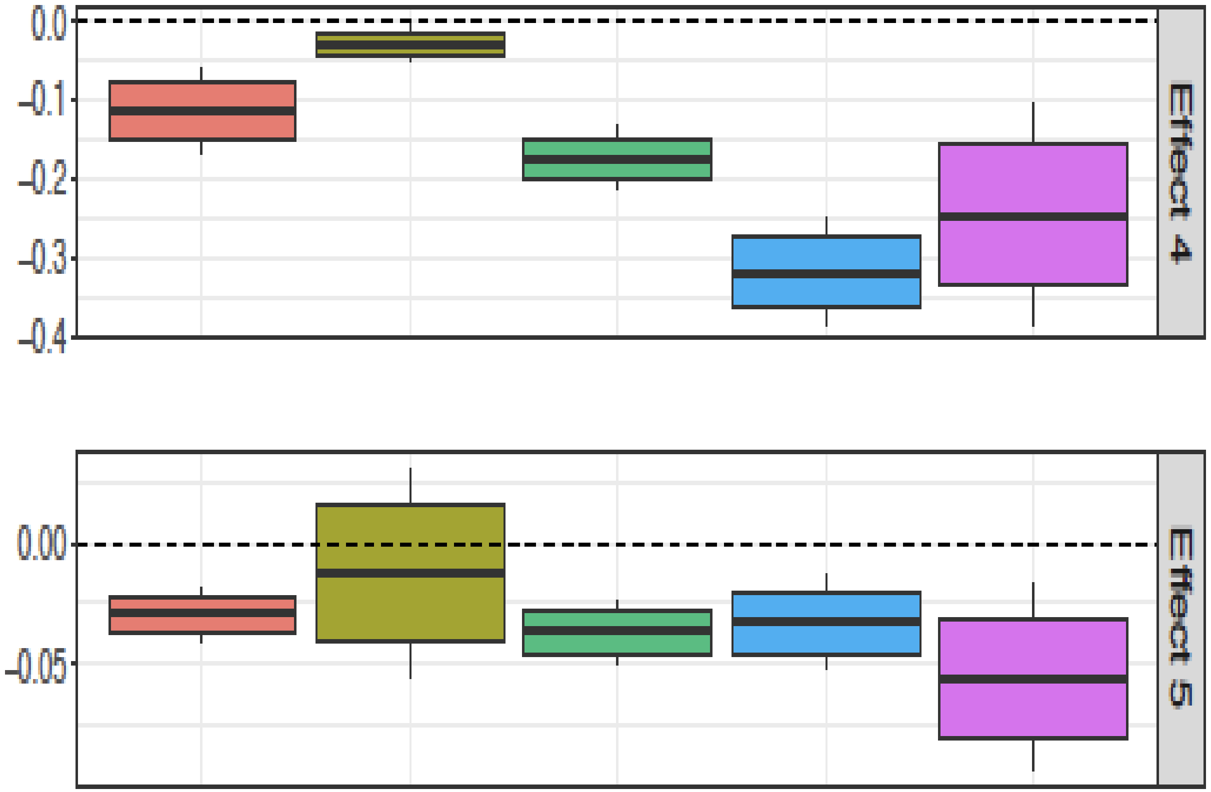


Effects Under Interference
Effect 1
Effect 2
Effect 3
Effect 4
Effect 5
0.0
−0.1
−0.2
−0.3
0.00
−0.02
−0.04
−0.06
0.02
0.01
0.00
−0.01
−0.02
0.0
−0.1
−0.2
−0.3
−0.4
0.00
−0.05
Region
USA
MIDWEST
NORTHEAST
SOUTHEAST
WEST



Interpretation of endogenous effects

- τ_1 (A$\rightarrow Y$): Democratic-leaning counties consistently show lower COVID-19 mortality across all Regions
- τ_2 (A $\rightarrow M \rightarrow Y$): Lower vaccine hesitancy in Democratic-leaning counties mediates the reduction in COVID-19 mortality



Interpretation of spillover effects

- $\tau_4(A^f \rightarrow Y)$: Neighboring D-leaning counties contribute to reduced COVID-19 mortality in adjacent counties.
- $\tau_5(A^f \rightarrow M \rightarrow Y)$: D-leaning neighbors influence local vaccine hesitancy, indirectly reducing COVID-19 mortality.


FOX

Concluding Remarks

- A rigorous analytic framework to investigate causal spillover effects, which offers important new insights in the analysis of network or environment dependent data.
- Extend the existing sequential ignorability conditions for the identifiability of causal mediation and spillover effects.
- Theoretical justification under non-iid samples is provided.
- The use of instrumental variables to handle unmeasured confounding is an important future work.

Thanks for your Attention!