Regularized Fine-Tuning for Representation Multi-Task Learning: Adaptivity, Minimaxity, and Robustness

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Joint work with

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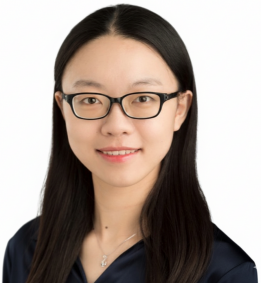
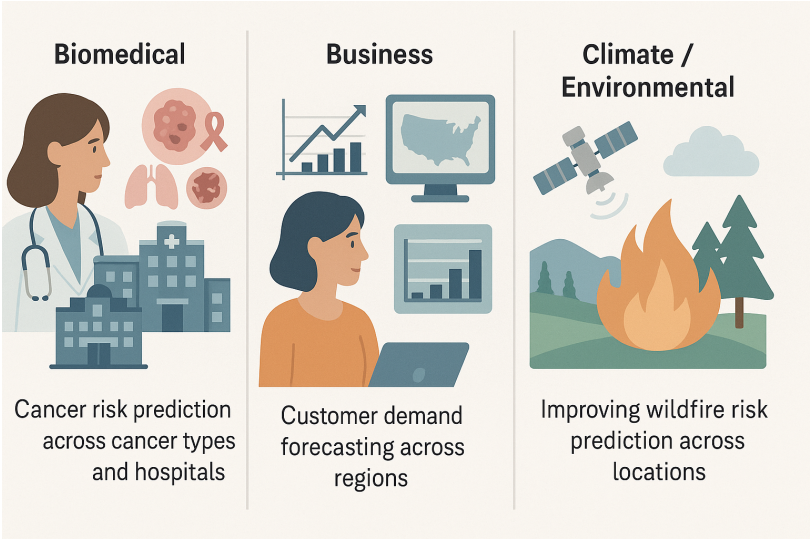


Yuqi Gu
(Columbia University)

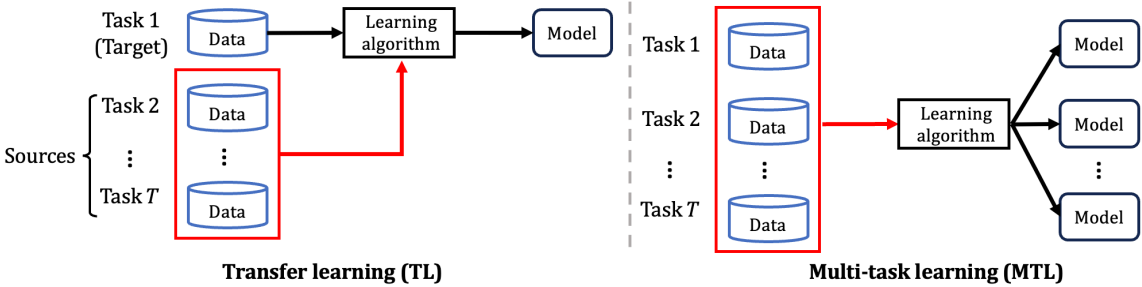
Outline

§1 Introduction

Motivation: Data Integration Across Multiple Sources



Knowledge transfer



- Borrowing information across tasks can reduce variance and improve generalization.
- However, task heterogeneity or corrupted data (outlier tasks) can degrade performance.
- **Goal:** develop adaptive and robust methods to transfer knowledge effectively under heterogeneity and possible task-level contamination.

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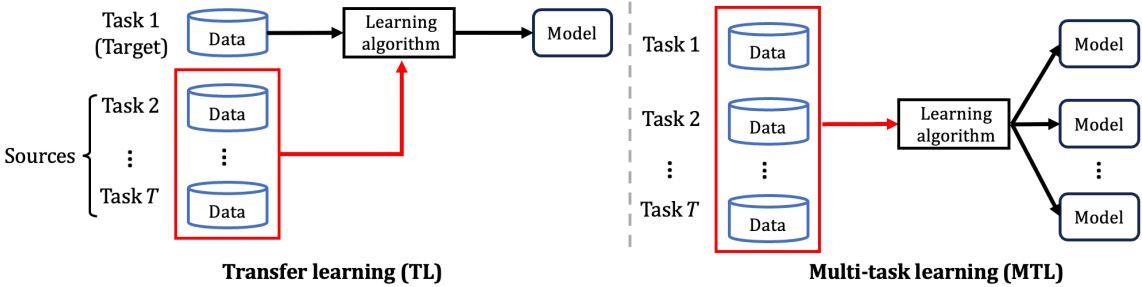

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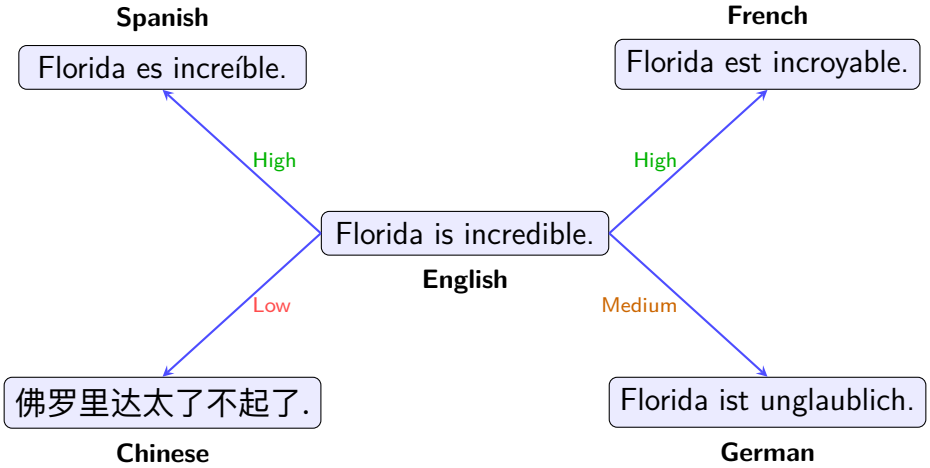
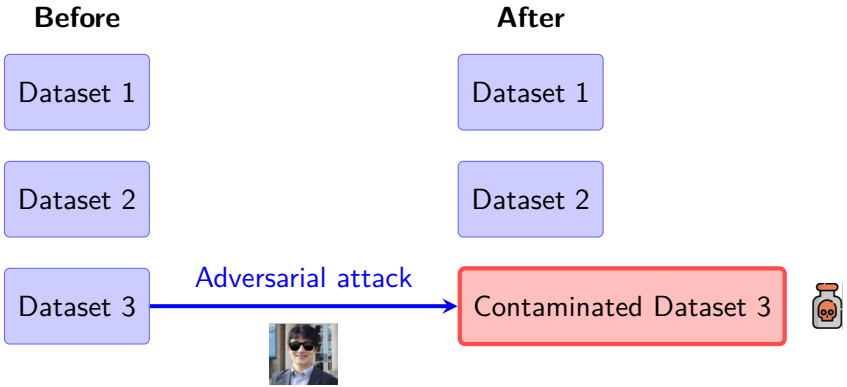
Challenge 1: Unknown Similarity



Challenge 2: Data Contamination



- **Today:** how to address them in **representation transfer/multi-task learning**

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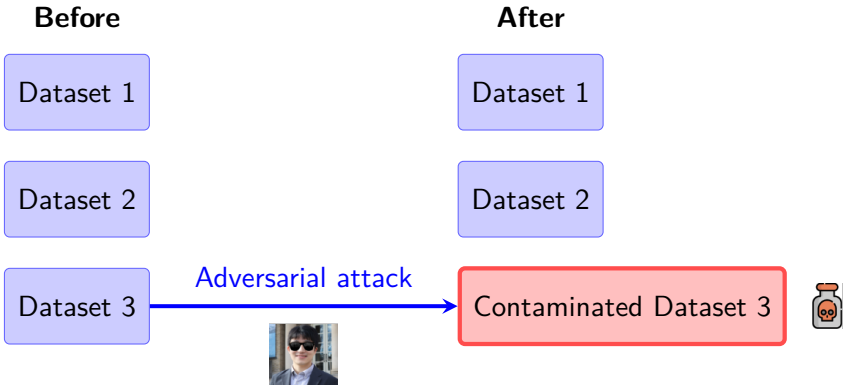

- **Today:** how to address them in **representation transfer/multi-task learning**

Background: What is Representation Learning?

- **Goal:** Learn a compact, informative feature subspace that summarizes complex, high-dimensional data.
- Instead of analyzing each dataset or task separately, we discover a **shared representation** that captures the underlying common structure.
- **Examples:**
  - In **biomedicine**: integrating gene expression and imaging data to predict *cancer risk* across institutions.
  - In **business analytics**: learning customer embeddings that generalize across *regions or product lines*.
  - In **environmental science**: deriving latent features from satellite and sensor data for *climate risk prediction*.
- Such representations enable **transfer and generalization**, allowing related tasks to learn collaboratively and adapt under heterogeneity.

§2 Setup, Algorithms, and Theory

Setup: Representation multi-task learning

- We observe $\{\boldsymbol{x}_i^{(t)}, y_i^{(t)}\}_{i=1}^n$ from the t-th task:

$$y_i^{(t)} = g^{(t)}\left(f^{(t)}(\boldsymbol{x}_i^{(t)})\right) + \epsilon_i^{(t)}, \quad i = 1:n, \quad t = 1:T,$$

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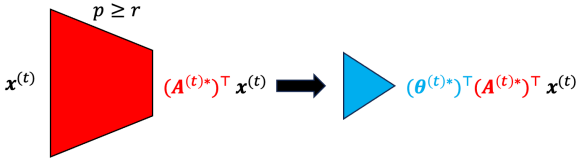
- In this talk:
  - linear representation $f^{(t)}(\boldsymbol{x}) = (\boldsymbol{A}^{(t)*})^\top \boldsymbol{x}$, $\boldsymbol{A}^{(t)*} \in \mathcal{O}^{p\times r}$ [1].
  - linear downstream model $g^{(t)}(\boldsymbol{z}) = \boldsymbol{z}^\top \boldsymbol{\theta}^{(t)*}$, $\boldsymbol{\theta}^{(t)*} \in \mathbb{R}^r$



- In other words: $y_i^{(t)} = (\boldsymbol{\beta}^{(t)*})^\top \boldsymbol{x}_i^{(t)} + \epsilon_i^{(t)}$, with $\boldsymbol{\beta}_{p\times 1}^{(t)*} = \boldsymbol{A}_{p\times r}^{(t)*} \boldsymbol{\theta}_{r\times 1}^{(t)*}$.

[1] $\mathcal{O}^{p\times r} := \{\boldsymbol{A} \in \mathbb{R}^{p\times r} : \boldsymbol{A}^\top \boldsymbol{A} = \boldsymbol{I}_r\}$

Setup: Representation multi-task learning

- **Unknown similarity:** $\boldsymbol{\beta}^{(t)*} = \boldsymbol{A}^{(t)*}\boldsymbol{\theta}^{(t)*}$ with

$$\min_{\overline{\boldsymbol{A}} \in \mathcal{O}^{p \times r}} \max_{t \in [T]} \|\boldsymbol{A}^{(t)*}(\boldsymbol{A}^{(t)*})^\top - \overline{\boldsymbol{A}}(\overline{\boldsymbol{A}})^\top\|_2 \leq h, \text{ where } \overline{\boldsymbol{A}} \text{ is an "average" representation.}$$

- **Task-level ϵ-contamination:** Konstantinov and Lampert (2019); Konstantinov et al. (2020)

  The attacker picks $S^c \subseteq [T]$ with $|S^c|/T \leq \epsilon$, for $t \in S^c$, :

$$\{\boldsymbol{x}_i^{(t)}, y_i^{(t)}\}_{i=1}^n \xrightarrow{\text{attacker}} \{\widetilde{\boldsymbol{x}}_i^{(t)}, \widetilde{y}_i^{(t)}\}_{i=1}^n$$

  - A stronger "MTL version" of Huber contamination (Huber, 1964)

- **Goal:** Learn uncontaminated $\{\boldsymbol{\beta}^{(t)*}\}_{t \in S}$, $S = [T] \backslash S^c$

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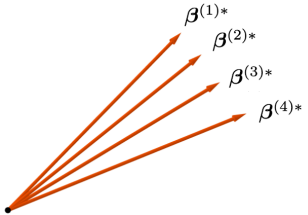
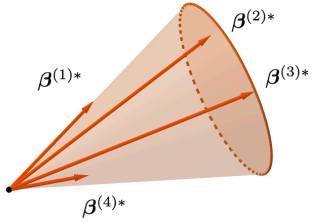
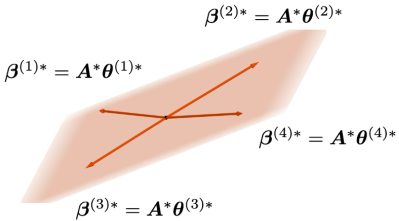
Related works: other parametric knowledge transfer



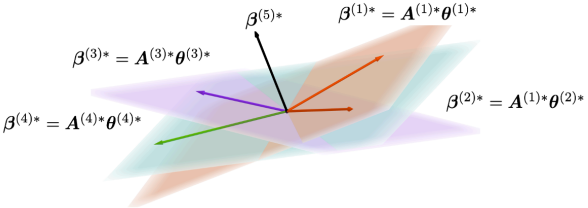
(a) Distance-based similarity



(b) Angle-based similarity

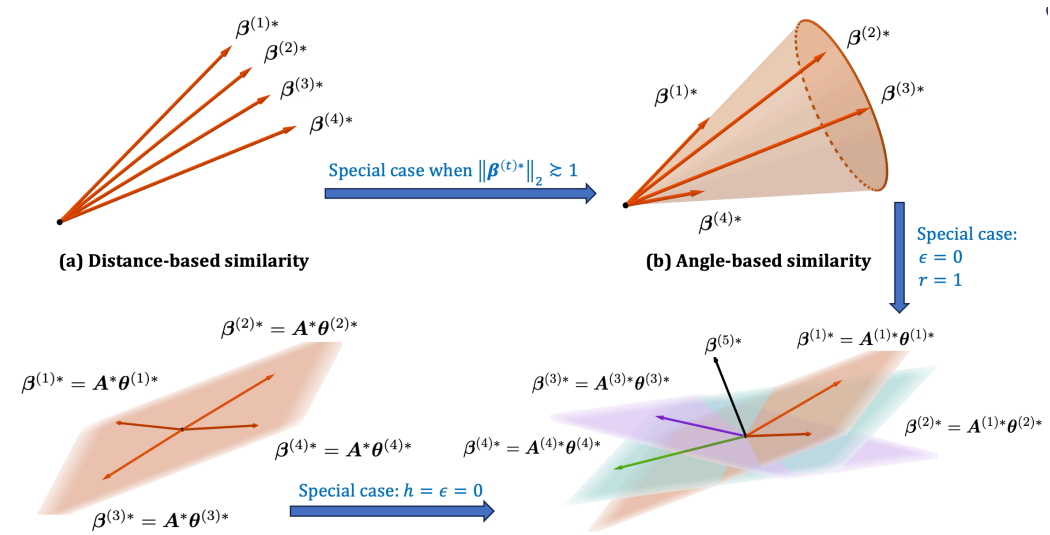


(c) Sharing a representation

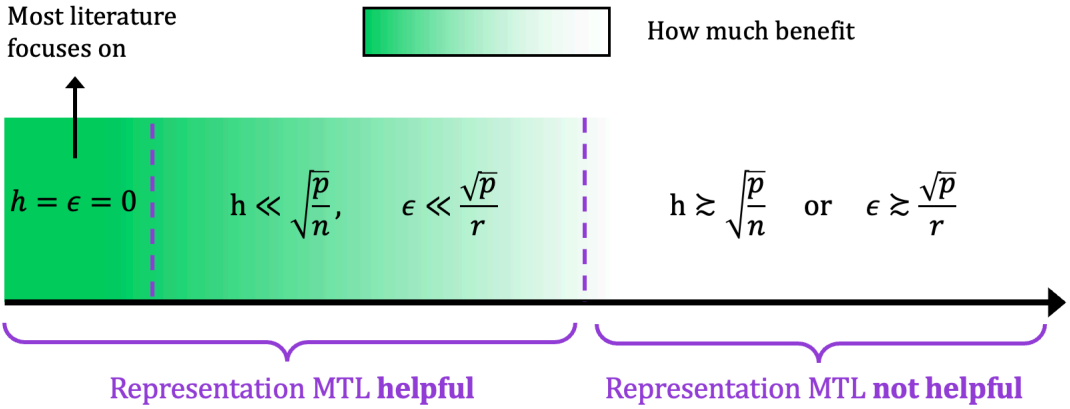


(d) Our setting

Related works: other parametric knowledge transfer



Different Regimes



Algorithm 1: penalized ERM

Algorithm 1 (Penalized ERM): Given $\lambda, \gamma > 0$

- **Step 1: (Pooling)** $\widehat{\boldsymbol{A}}^{(t)}, \widehat{\boldsymbol{\theta}}^{(t)}, \widehat{\overline{\boldsymbol{A}}} \leftarrow \min_{\boldsymbol{A}^{(t)}, \overline{\boldsymbol{A}} \in \mathcal{O}^{p\times r}, \boldsymbol{\theta}^{(t)} \in \mathbb{R}^r} \sum_{t=1}^{T} \left\{ \frac{1}{n} \sum_{i=1}^{n} [y_i^{(t)} - (\boldsymbol{x}^{(t)})^\top \boldsymbol{A}^{(t)} \boldsymbol{\theta}^{(t)}]^2 + \frac{\lambda}{\sqrt{n}} \| \boldsymbol{A}^{(t)} (\boldsymbol{A}^{(t)})^\top - \overline{\boldsymbol{A}} (\overline{\boldsymbol{A}})^\top \|_2 \right\}$
- **Step 2: (Regularized Fine-Tuning)** $\widehat{\boldsymbol{\beta}}^{(t)} \leftarrow \min_{\boldsymbol{\beta} \in \mathbb{R}^p} \left\{ \frac{1}{n} \sum_{i=1}^{n} [y_i^{(t)} - (\boldsymbol{x}^{(t)})^\top \boldsymbol{\beta}]^2 + \frac{\gamma}{\sqrt{n}} \| \boldsymbol{\beta} - \widehat{\boldsymbol{A}}^{(t)} \widehat{\boldsymbol{\theta}}^{(t)} \|_2 \right\}$

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Theorem 1 (Estimation error of pERM)

(i) When $\lambda \gtrsim \sqrt{r(p+\log T)}$, $\gamma \asymp \sqrt{p+\log T}$*: up to log-terms, w.h.p.*

$$\max_{t\in S} \|\widehat{\boldsymbol{\beta}}^{(t)} - \boldsymbol{\beta}^{(t)*}\|_2 \lesssim \Bigg(\underbrace{r\sqrt{\frac{p}{nT}}}_{\text{learn } \boldsymbol{A}^{(t)*}} + \underbrace{\sqrt{r}h}_{\boldsymbol{A}^{(t)*} \text{not equal}} + \underbrace{r\sqrt{\frac{1}{n}}}_{\text{learn } \boldsymbol{\theta}^{(t)*}} + \underbrace{\frac{\lambda\epsilon r}{\sqrt{n}}}_{\text{outlier tasks}} \Bigg) \wedge \underbrace{\sqrt{\frac{p}{n}}}_{\text{single-task rate}}$$

(ii) If tasks in S^c *also follow linear model: up to log-terms, w.h.p.*

$$\max_{t\in S^c} \|\widehat{\boldsymbol{\beta}}^{(t)} - \boldsymbol{\beta}^{(t)*}\|_2 \lesssim \sqrt{\tfrac{p}{n}}.$$

- No contamination ($\epsilon = 0$)
  - Taking $\lambda = +\infty$ reduces to the ERM in Du et al. (2020); Tripuraneni et al. (2021)
  - No "bad" local minimizers
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Algorithm 2: spectral method

- When $h = 0$: $\boldsymbol{B}^* = \{\boldsymbol{\beta}^{(t)*}\}_{t \in S} \in \text{Col}(\overline{\boldsymbol{A}})$
  - SVD should work
  - Need to estimate $\boldsymbol{B}^*$ in a robust way

Algorithm 2 (Spectral method): Given $\gamma, R > 0$, an upper bound $\bar{\epsilon}$ (for ϵ)

- **Step 1: (Single-task regression)** $\widetilde{\boldsymbol{\beta}}^{(t)} = \text{OLS}$ on task t
- **Step 2: (Projection and concatenation)** $\widehat{\boldsymbol{B}} = (\prod_R(\widetilde{\boldsymbol{\beta}}^{(t)}) \ \ldots \ \prod_R(\widetilde{\boldsymbol{\beta}}^{(T)}))$, where $R = \texttt{quantile}(\{\|\widetilde{\boldsymbol{\beta}}^{(t)}\|_2\}_{t=1}^T, 1 - \bar{\epsilon})$.
- **Step 3: (SVD)** $\widehat{\widehat{\boldsymbol{A}}} =$ left sing. matrix of $\widehat{\boldsymbol{B}}$, $\widehat{\boldsymbol{\theta}}^{(t)} = \text{OLS}$ in $\text{Col}(\widehat{\widehat{\boldsymbol{A}}})$ on task t
- **Step 4: (Biased regularization)**

$$\widehat{\boldsymbol{\beta}}^{(t)} \leftarrow \min_{\boldsymbol{\beta} \in \mathbb{R}^p} \left\{ \frac{1}{n} \sum_{i=1}^n [y_i^{(t)} - (\boldsymbol{x}^{(t)})^T \boldsymbol{\beta}]^2 + \frac{\gamma}{\sqrt{n}} \|\boldsymbol{\beta} - \widehat{\boldsymbol{A}}^{(t)} \widehat{\boldsymbol{\theta}}^{(t)}\|_2 \right\}$$

Algorithm 2: spectral method

Theorem 2 (Estimation error of the spectral method)

(i) With a proper $R > 0$, when $\gamma \asymp \sqrt{p + \log T}$: up to log-terms, w.h.p.

$$\max_{t \in S} \|\widehat{\boldsymbol{\beta}}^{(t)} - \boldsymbol{\beta}^{(t)*}\|_2 \lesssim \Bigg(\underbrace{\sqrt{\frac{pr}{nT}}}_{\text{learn } \boldsymbol{A}^{(t)*}} + \underbrace{h}_{\boldsymbol{A}^{(t)*} \text{ not equal}} + \underbrace{\sqrt{\frac{r}{n}}}_{\text{learn } \boldsymbol{\theta}^{(t)*}} + \underbrace{\sqrt{\epsilon r}}_{\text{outlier tasks}} \Bigg) \wedge \underbrace{\sqrt{\frac{p}{n}}}_{\text{single-task rate}}$$

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$$\max_{t \in S^c} \|\widehat{\boldsymbol{\beta}}^{(t)} - \boldsymbol{\beta}^{(t)*}\|_2 \lesssim \sqrt{\frac{p}{n}}.$$

- Polynomial-time algorithm
- Better than the bound of pERM when $\epsilon = 0$
- Minimax optimal when $\epsilon = 0$

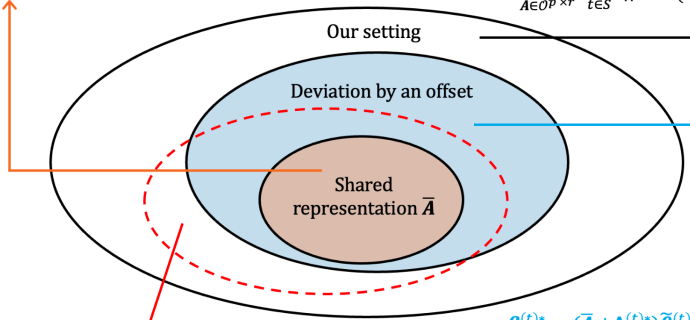
Comparing with exisiting methods

$\boldsymbol{\beta}^{(t)*} = \overline{\boldsymbol{A}}\boldsymbol{\theta}^{(t)*}, t \in S = [T]$ (Du et al., 2020; Thekumparampil et al., 2021; Tripuraneni et al., 2021)

$\max_{t\in[T]} ||\boldsymbol{\theta}^{(t)*}||_2 \lesssim 1$

$\boldsymbol{\beta}^{(t)*} = \boldsymbol{A}^{(t)*}\boldsymbol{\theta}^{(t)*}, \mathrm{t} \in S \subseteq [T]$

$\min_{\overline{\boldsymbol{A}}\in\mathcal{O}^{p\times r}} \max_{t\in S} ||\boldsymbol{A}^{(t)*}\left(\boldsymbol{A}^{(t)*}\right)^{\top} - \overline{\boldsymbol{A}}(\overline{\boldsymbol{A}})^{\top}||_2 \leq h$



$\boldsymbol{\beta}^{(t)*} = (\overline{\boldsymbol{A}} + \boldsymbol{\Delta}^{(t)*})\widetilde{\boldsymbol{\theta}}^{(t)*}, t \in S = [T]$

$\max_{t\in[T]} ||\widetilde{\boldsymbol{\theta}}^{(t)*}||_2 \lesssim 1$ (Chua et al., 2021)

$\boldsymbol{\beta}^{(t)*} = \overline{\boldsymbol{A}}\overline{\boldsymbol{\theta}}^{(t)*} + \boldsymbol{\delta}^{(t)*}, \mathrm{t} \in S = [T]$

$\max_{t\in[T]} ||\overline{\boldsymbol{\theta}}^{(t)*}||_2 \lesssim 1$ (Duan and Wang, 2023)

The regime for rate comparison in Table 1, our setting with $\max_{t\in S} ||\boldsymbol{\theta}^{(t)*}||_2 \lesssim 1, S \subseteq [T]$ and $\sqrt{|S|^{-1}\sum_{t\in S}||\boldsymbol{\theta}^{(t)*}||_2^2} \gtrsim 1$

Review: Comparison of estimation error

Regime	Algorithm	$\max_{t\in S}\|\widehat{\boldsymbol{\beta}}^{(t)}-\boldsymbol{\beta}^{(t)*}\|_2$	Optimal ($\epsilon=0$)?	Poly-time?
$h=\epsilon=0$	ERM (Du et al., 2020; Tripuraneni et al., 2021)	$r\sqrt{\frac{p}{nT}}+r\sqrt{\frac{1}{n}}$	No	No
	MoM (Tripuraneni et al., 2021)	$r\sqrt{\frac{p}{nT}}+\sqrt{\frac{r}{n}}$	No	Yes
	AltMinGD (Thekumparampil et al., 2021)	$r\sqrt{\frac{p}{nT}}+\sqrt{\frac{r}{n}}$	No	Yes
$h\neq 0$	AdaptRep (Chua et al., 2021)	$\left[r\sqrt{\frac{p}{nT}}+r\sqrt{\frac{1}{n}}+\sqrt{rh}(\frac{p}{n})^{1/4}\right]\wedge\sqrt{\frac{p}{n}}$	No	No
$\epsilon=0$	ARMUL (Duan and Wang, 2023)	$\left(r\sqrt{\frac{p}{nT}}+r\sqrt{\frac{1}{n}}+rh\right)\wedge\left(r\sqrt{\frac{p}{n}}\right)$	No	No
$h,\epsilon\neq 0$	pERM (Algorithm 1)	$\left(r\sqrt{\frac{p}{nT}}+r\sqrt{\frac{1}{n}}+\sqrt{r}h+\epsilon\frac{r^{3/2}\sqrt{p}}{\sqrt{n}}\right)\wedge\sqrt{\frac{p}{n}}$	No	No
	Spectral (Algorithm 2)	$\left(\sqrt{\frac{pr}{nT}}+\sqrt{\frac{r}{n}}+h+\sqrt{\epsilon r}\right)\wedge\sqrt{\frac{p}{n}}$	Yes	Yes
	Single-task	$\sqrt{\frac{p}{n}}$	No	Yes
	Lower-bound	$\left(\sqrt{\frac{pr}{nT}}+\sqrt{\frac{r}{n}}+h+\epsilon\frac{r}{\sqrt{n}}\right)\wedge\sqrt{\frac{p}{n}}$	–	–

§3 Simulation Studies

Simulation Settings

- Generate sample $\{\boldsymbol{x}_i^{(t)}, y_i^{(t)}\}_{i=1}^n$ from the t-th task, $t = 1 : T$, and

$$y_i^{(t)} = (\boldsymbol{x}_i^{(t)})^\top \boldsymbol{\beta}^{(t)*} + \epsilon_i^{(t)}, \quad i = 1 : n,$$

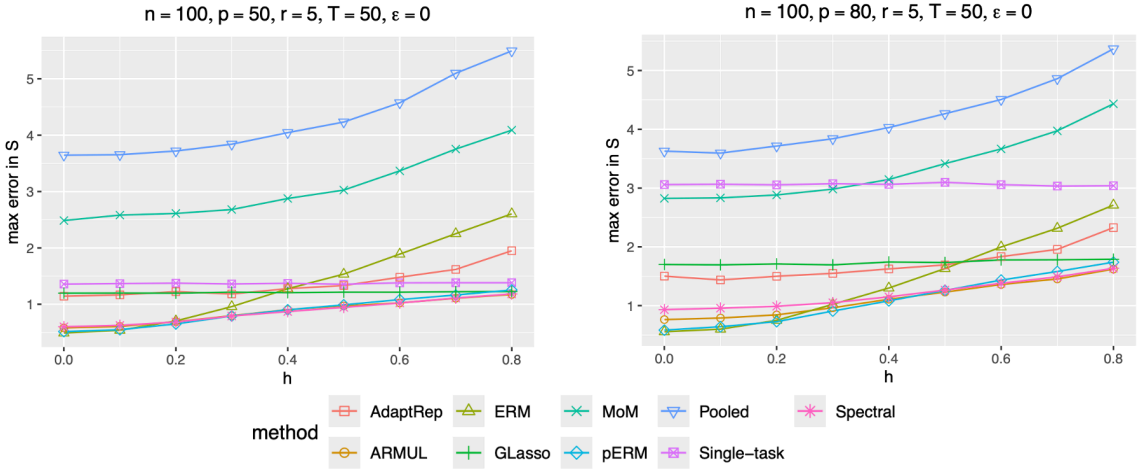
  where $\boldsymbol{\beta}^{(t)*} = \boldsymbol{A}^{(t)*}\boldsymbol{\theta}^{(t)*}$, $\boldsymbol{A}^{(t)*} \in \mathcal{O}^{p\times r}$, $\boldsymbol{\theta}^{(t)*} \in \mathbb{R}^r$.

- We generate
  - $\boldsymbol{x}_i^{(t)} \overset{i.i.d.}{\sim} N(\boldsymbol{0}_p, \boldsymbol{I}_p)$
  - $\epsilon_i^{(t)} \overset{i.i.d.}{\sim} N(0, 1)$
  - $p \times r$ matrix $\boldsymbol{C}$ with i.i.d. standard normal entries.
- We defined $\overline{\boldsymbol{A}}$ as the first r columns of the left singular matrix of $\boldsymbol{C}$, $\widetilde{\boldsymbol{A}}^{(t)} = \overline{\boldsymbol{A}} + a^{(t)}(\boldsymbol{I}_{r\times r}, \boldsymbol{0}_{r\times(p-r)})^\top$, and $\boldsymbol{A}^{(t)*} = \widetilde{\boldsymbol{A}}^{(t)}[(\widetilde{\boldsymbol{A}}^{(t)})^\top \widetilde{\boldsymbol{A}}^{(t)}]^{-1}(\widetilde{\boldsymbol{A}}^{(t)})^\top$ for $t \in [T]$, where $a^{(t)}$'s are i.i.d. sampled from Unif$([-h, h])$.
- We generated each coordinate of $\boldsymbol{\theta}^{(t)*} \in \mathbb{R}^r$ from Unif$([-2, 2])$ independently.
- 100 replicates

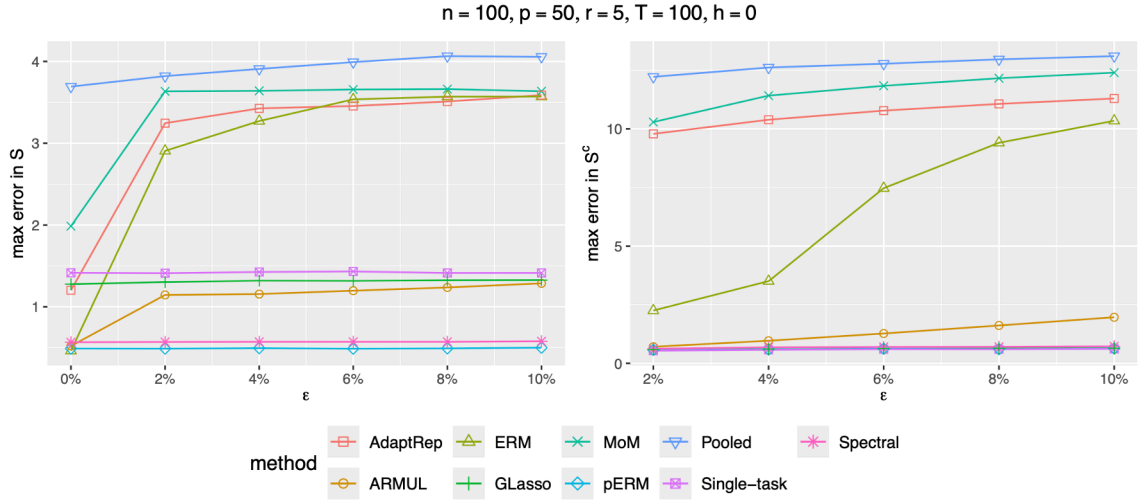
Implementation and benchmarks

- We used the Adam solver (Kingma and Ba, 2015) in PyTorch to solve the optimization problems
- Our methods:
  - Penalized ERM
  - Spectral method
- Benchmarks:
  - Single-task regression
  - Pooled regression (Crammer et al., 2008)
  - ERM (Du et al., 2020; Tripuraneni et al., 2021)
  - Method-of-moments (MoM) (Tripuraneni et al., 2021)
  - ARMUL (Duan and Wang, 2023)
  - AdaptRep (Chua et al., 2021)
  - Group Lasso (GLasso) (Lounici et al., 2011)

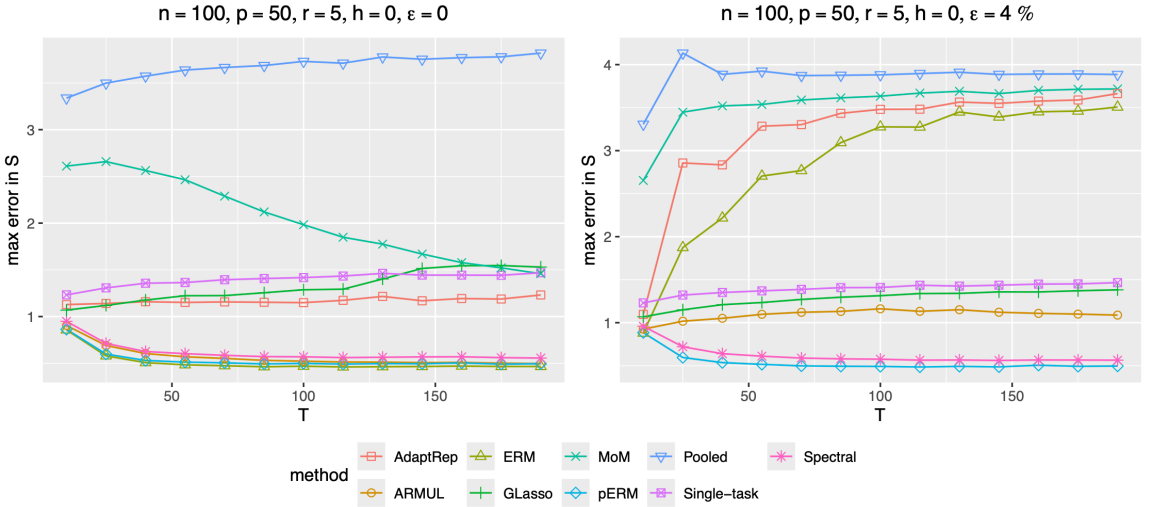
Different heterogeneity parameter h



Different contamination proportion ϵ



Different number of tasks T



§4 Human Activity Recognition Dataset Analysis

Human Activity Recognition Dataset

- The data come from **30 volunteers** performing six activities (walking, walking upstairs, walking downstairs, sitting, standing, and lying) while carrying a smartphone (Anguita et al., 2013).[2]
- Each observation contains $p = 561$ time- and frequency-domain features extracted from smartphone sensor signals.
- We treat each volunteer as a **separate task**, with sample sizes per task ranging from **281 to 409**.
- We focus on a **binary classification** problem: distinguishing *active postures* (walking, walking upstairs, walking downstairs, standing) from *inactive postures* (sitting, lying).
- For each task, in each of the **100 replicates**, 50% of the samples are used for training and 50% for testing.
- We report the **average test misclassification error rates** across all $T = 30$ **tasks** under varying values of r.

[2] Available at the UCI Machine Learning Repository: `https://archive.ics.uci.edu/ml/datasets/human+activity+recognition+using+smartphones`.

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Mis-classification test error rates

r/Method	Single-task	Pooled	ERM	ARMUL	pERM	Spectral	GLasso
$r = 5$	1.66 (0.20)	1.79 (0.21)	1.62 (1.42)	2.12 (0.27)	**1.33 (0.23)**	1.85 (0.27)	1.44 (0.25)
$r = 10$	1.66 (0.20)	1.79 (0.21)	1.42 (0.23)	1.77 (0.23)	**1.25 (0.20)**	1.47 (0.18)	1.44 (0.25)
$r = 15$	1.66 (0.20)	1.79 (0.21)	1.36 (0.23)	1.68 (0.21)	**1.36 (1.07)**	1.50 (0.19)	1.44 (0.25)

Table: The average mis-classification test error rates (standard deviations) of different methods over $T = 30$ tasks with different r values. All values are in percentages.

§5 Take-away

Take-away

- **Robust and adaptive representation learning** is essential for integrating heterogeneous data across domains.
- **Theoretical guarantees** provide rigor and reliability even under contamination and unknown similarity.
- **Future directions:** nonlinear situation, scaling to multimodal and dynamic settings such as EHR, genomics, and imaging.

- *Tian, Y., Gu, Y., & Feng, Y. (2025). Learning from Similar Linear Representations: Adaptivity, Minimaxity, and Robustness. JMLR.*

My other works in transfer learning/multi-task learning

- **Transfer Learning in High-Dimensional GLMs (JASA, 2023)**
  Y. Tian & Y. Feng
- **Theory of Unsupervised Federated Learning (ICML, 2024)**
  Y. Tian, H. Weng, & Y. Feng
- **Unsupervised Multi-Task and Transfer Learning on GMMs (arXiv:2209.15224, 2024)**
  Y. Tian, H. Weng, L. Xia, & Y. Feng
- **Federated Transfer Learning with Differential Privacy (arXiv:2403.11343, 2024)**
  M. Li, Y. Tian, Y. Feng, & Y. Yu
- **GeoERM: Geometry-Aware Multi-Task Learning (arXiv:2505.02972, 2025)**
  A. Chen & Y. Feng

SHORT COURSE on TL

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merci
suksema
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baie dankie
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tänan
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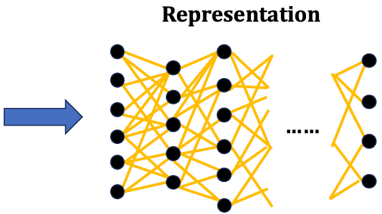
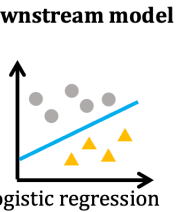
Representation transfer/multi-task learning

ImageNet
~1.3M training images

Representation transfer/multi-task learning

ImageNet
~1.3M training images

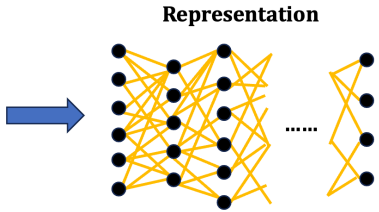




Representation transfer/multi-task learning

ImageNet
~1.3M training images

A much smaller dataset



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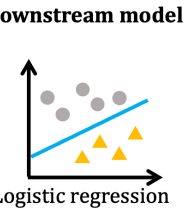
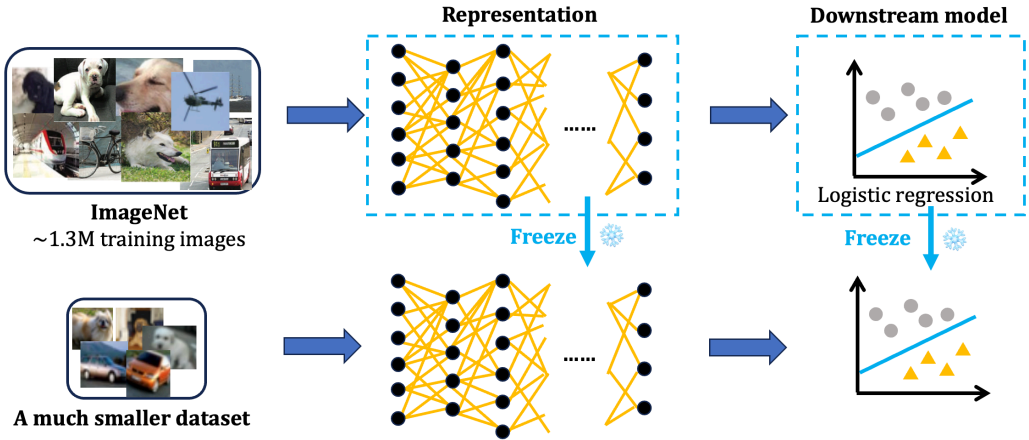
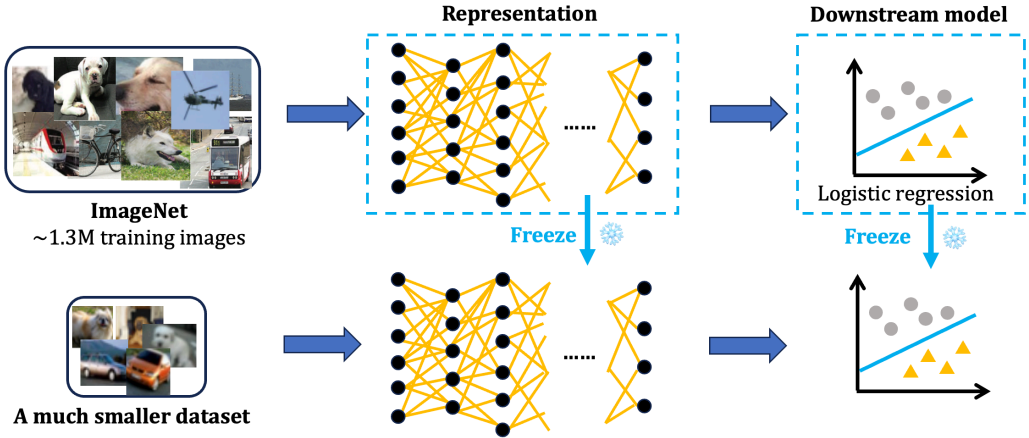


Representation transfer/multi-task learning

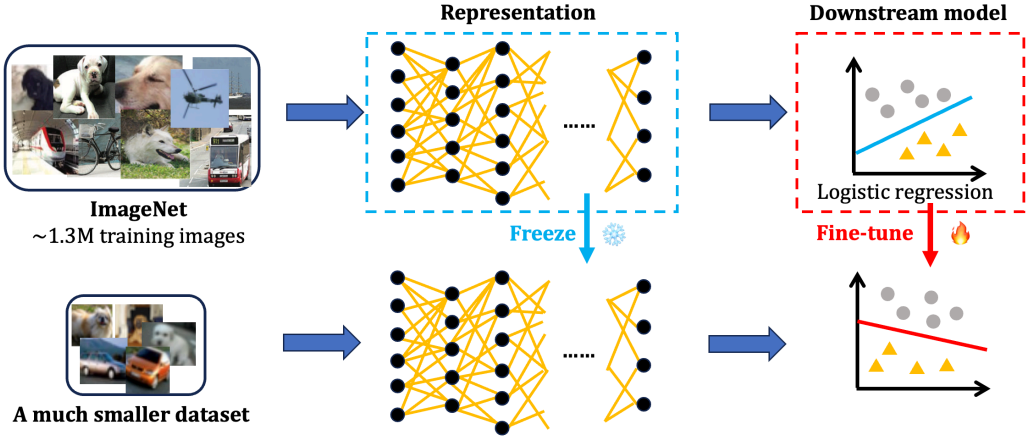


Representation transfer/multi-task learning

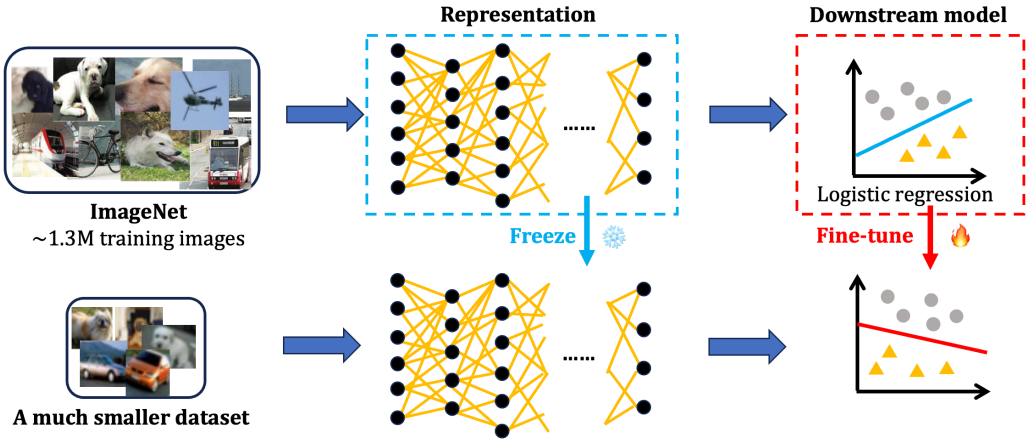


❄ representation + ❄ downstream models: not a good idea

Representation transfer/multi-task learning

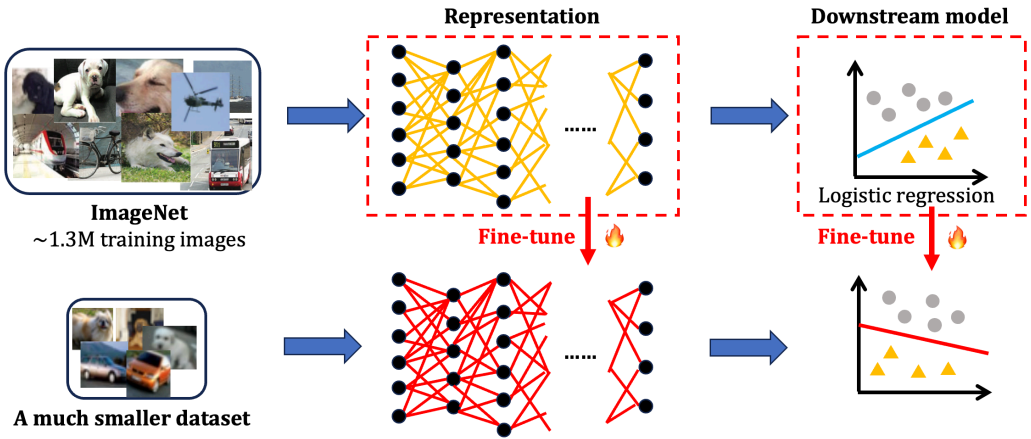


Representation transfer/multi-task learning

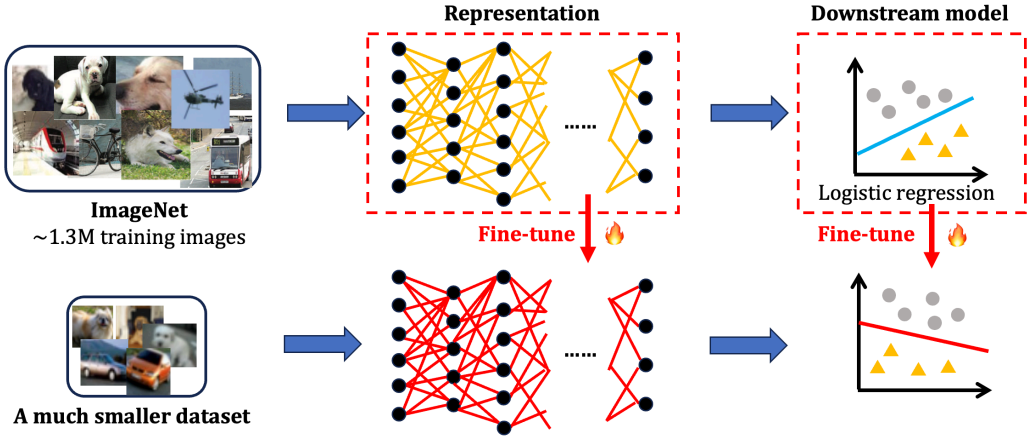


❄ representation + 🔥 downstream models (Donahue et al., 2014): can lead to negative transfer (Kornblith et al., 2019).

Representation transfer/multi-task learning



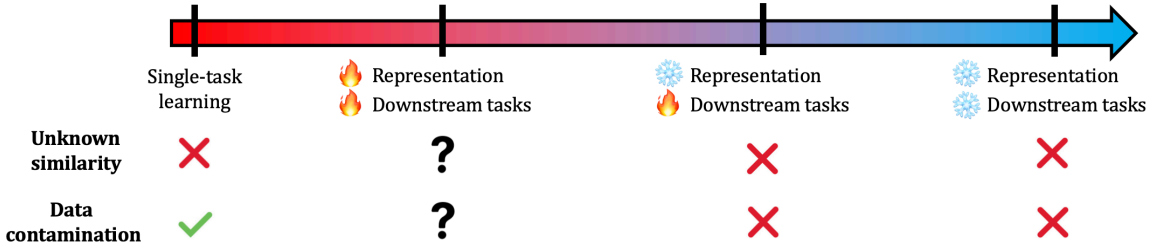
Representation transfer/multi-task learning



🔥 representation + 🔥 downstream models (Kornblith et al., 2019): sometimes unstable, "catastrophic forgetting" (Lee et al., 2020)

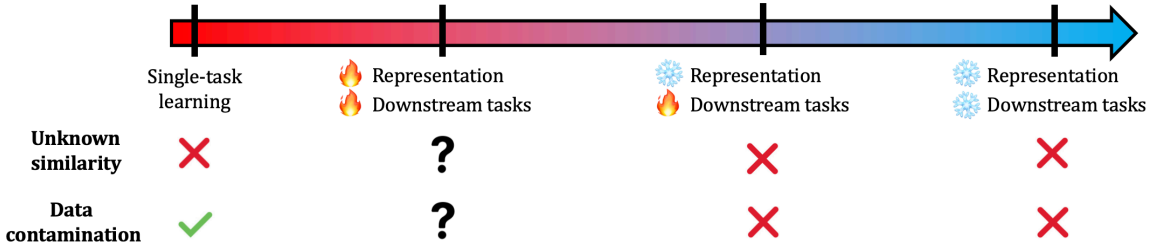
A quick summary

- Different approaches and two challenges:



A quick summary

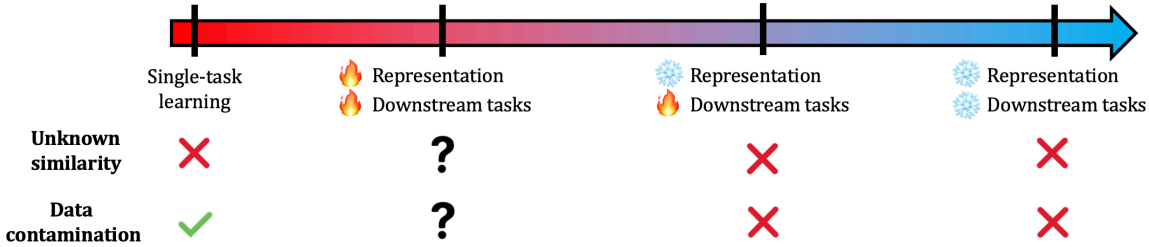
- Different approaches and two challenges:



- **Goal 1:** Show how **regularized fine-tuning** can replace both **?** with ✓

A quick summary

- Different approaches and two challenges:



- **Goal 1:** Show how **regularized fine-tuning** can replace both **?** with ✓
- **Goal 2:** Quantify the impact of two challenges on representation transfer/multi-task learning.